LTL Syntax: Context-Free Grammar

$$
\begin{array}{lcll}
\phi & ::= & \top & [\ \textit{true}\ ] \\
 & | & \bot & [\ \textit{false}\ ] \\
 & | & p & [\ \texttt{propositional atom}\ ] \\
 & | & (\neg\phi) & [\ \texttt{logical negation}\ ] \\
 & | & (\phi \wedge \phi) & [\ \texttt{logical conjunction}\ ] \\
 & | & (\phi \vee \phi) & [\ \texttt{logical disjunction}\ ] \\
 & | & (\phi \Rightarrow \phi) & [\ \texttt{logical implication}\ ] \\
 & | & (\mathbf{X}\,\phi) & [\ \texttt{neXt state}\ ] \\
 & | & (\mathbf{F}\,\phi) & [\ \texttt{some Future state}\ ] \\
 & | & (\mathbf{G}\,\phi) & [\ \texttt{all future states (Globally)}\ ] \\
 & | & (\phi\,\mathbf{U}\,\phi) & [\ \texttt{Until}\ ] \\
 & | & (\phi\,\mathbf{W}\,\phi) & [\ \texttt{Weak-until}\ ] \\
 & | & (\phi\,\mathbf{R}\,\phi) & [\ \texttt{Release}\ ]
\end{array}
$$

Parsing: Some Practical Knowledge

$$
\begin{array}{lcll}
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\end{array}
$$

Assumption: Operator **precedence** considered first before the CFG.

Interpreting a Formula: Parse Trees (1)

$$
\begin{array}{lll}
\phi & ::= & \top \quad [\textit{true}] \\
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 & | & (\mathbf{G}\phi) \quad [\text{all future states (}\mathbf{G}\text{lobally)}] \\
 & | & (\phi\,\mathbf{U}\,\phi) \quad [\mathbf{U}\text{ntil}] \\
 & | & (\phi\,\mathbf{W}\,\phi) \quad [\mathbf{W}\text{eak-until}] \\
 & | & (\phi\,\mathbf{R}\,\phi) \quad [\mathbf{R}\text{elease}]
\end{array}
$$

$$\mathbf{F}\ p \wedge \mathbf{G}\ q \Rightarrow p\ \mathbf{U}\ r$$

Interpreting a Formula: Parse Trees (2)

$$
\begin{array}{lll}
\phi & ::= & \top \quad [\textit{true}] \\
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\end{array}
$$

$$\mathbf{F}\,(p \wedge \mathbf{G}\,q \Rightarrow p\,\mathbf{U}\,r)$$

Interpreting a Formula: Parse Trees (3)

$$
\begin{array}{lll}
\phi & ::= & \top \quad [\textit{true}] \\
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\end{array}
$$

$$\mathbf{F}\ p \wedge (\mathbf{G}\ q \Rightarrow p\ \mathbf{U}\ r)$$

Interpreting a Formula: Parse Trees (4)

$$
\begin{array}{lll}
\phi & ::= & \top \quad [\textit{true}] \\
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\end{array}
$$

$$\mathbf{F}\ p \wedge ((\mathbf{G}\ q \Rightarrow p)\ \mathbf{U}\ r)$$

Interpreting a Formula: LMD (1)

ϕ	::=	$\top$	[*true*]
	|	$\bot$	[*false*]
	|	p	[propositional atom]
	|	$(\neg\phi)$	[logical negation]
	|	$(\phi \wedge \phi)$	[logical conjunction]
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	|	$(\phi \Rightarrow \phi)$	[logical implication]
	|	$(\mathbf{X}\phi)$	[ne**X**t state]
	|	$(\mathbf{F}\phi)$	[some **F**uture state]
	|	$(\mathbf{G}\phi)$	[all future states (**G**lobally)]
	|	$(\phi\,\mathbf{U}\,\phi)$	[**U**ntil]
	|	$(\phi\,\mathbf{W}\,\phi)$	[**W**eak-until]
	|	$(\phi\,\mathbf{R}\,\phi)$	[**R**elease]

$$\mathbf{F}\,p \wedge \mathbf{G}\,q \Rightarrow p\,\mathbf{U}\,r$$

Interpreting a Formula: LMD (2)

$$
\begin{array}{lll}
\phi & ::= & \top \quad [\textit{true}] \\
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 & | & (\neg\phi) \quad [\texttt{logical negation}] \\
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\end{array}
$$

$$\mathbf{F}\,(p \wedge \mathbf{G}\,q \Rightarrow p\,\mathbf{U}\,r)$$

Interpreting a Formula: LMD (3)

$$
\begin{array}{lcll}
\phi & ::= & \top & [\,\textit{true}\,] \\
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 & | & p & [\,\texttt{propositional atom}\,] \\
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 & | & (\phi\,\mathbf{U}\,\phi) & [\,\mathbf{U}\texttt{ntil}\,] \\
 & | & (\phi\,\mathbf{W}\,\phi) & [\,\mathbf{W}\texttt{eak-until}\,] \\
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\end{array}
$$

$$\mathbf{F}\ p \wedge (\mathbf{G}\ q \Rightarrow p\ \mathbf{U}\ r)$$

Interpreting a Formula: LMD (4)

$$
\begin{array}{lll}
\phi & ::= & \top \quad [\textit{true}] \\
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\end{array}
$$

$$\mathbf{F}\ p \wedge ((\mathbf{G}\ q \Rightarrow p)\ \mathbf{U}\ r)$$

Interpreting a Formula: RMD (1)

ϕ	::=	$\top$	[*true*]
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$$\mathbf{F}\,p \wedge \mathbf{G}\,q \Rightarrow p\,\mathbf{U}\,r$$

Interpreting a Formula: RMD (2)

ϕ	::=	$\top$	[*true*]
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	|	p	[propositional atom]
	|	$(\neg\phi)$	[logical negation]
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$$\mathbf{F}\,(p \wedge \mathbf{G}\,q \Rightarrow p\,\mathbf{U}\,r)$$

Interpreting a Formula: RMD (3)

$$
\begin{array}{lcll}
\phi & ::= & \top & [\,\textit{true}\,] \\
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\end{array}
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$$\mathbf{F}\ p \wedge (\mathbf{G}\ q \Rightarrow p\ \mathbf{U}\ r)$$

Interpreting a Formula: RMD (4)

ϕ	::=	$\top$	[*true*]
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$$\mathbf{F}\ p \wedge ((\mathbf{G}\ q \Rightarrow p)\ \mathbf{U}\ r)$$

Interpreting a Formula: PT vs. LMD vs. RMD

$$\mathbf{F}\ p \wedge \mathbf{G}\ q \Rightarrow p\ \mathbf{U}\ r$$

Deriving Subformulas from a Parse Tree

Enumerate all subformulas of:

$$\mathbf{F}\ (p \Rightarrow \mathbf{G}\ r) \vee ((\neg\ q)\ \mathbf{U}\ p)$$

Labelled Transition System (LTS)

$M = (S, \longrightarrow, L)$, given P

Q. Formulate **deadlock freedom**:

From any state, it is always possible to make progress.

Labelled Transition System (LTS)

Exercises Consider the system with a counter c with the following assumption:

$$0 \leq c \leq 3$$

Say c is initialized 0 and may be incremented (via a transition *inc*, enabled when $c < 3$) or decremented (via a transition *dec*, enabled when $c > 0$).

- **Draw** a ***state graph*** of this system.
- **Formulate** the state graph as an ***LTS*** (via a triple $(S, \longrightarrow, L)$).

  Assume: Set P of atoms is: $\{\ c \geq 1, c \leq 1\ \}$

Labelled Transition System (LTS): Formulation & Paths